Questions: ① If $v, w \in \mathbb{R}^3$ and $w \times v = 0$, what does that mean about the two vectors?

② (from last time)

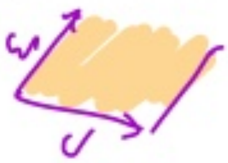
Find the equation of the plane that is parallel to the line

$$L(t) = (4+t,\ 7-t,\ 2+2t) = \underbrace{(4,7,2)}_{\text{a pt on line}} + t\underbrace{(1,-1,2)}_{\text{velocity}}$$

And contains the line

$$S(t) = (2-t,\ 4,\ t+1) = (2,4,1) + t(-1,0,1)$$

---

① Since $|w \times v| = |w|\,|v| \sin\theta \overset{\pm}{=} \text{area}$ 

If the result is $0 \iff \text{area} = 0$

$\iff$ v & w are collinear (along the same line)

$\iff$ v & w are parallel.

---

② $S(t)$ is on the plane $\Rightarrow$ $p = (2,4,1)$ is on the plane.

The vectors $(1,-1,2)$ & $(-1,0,1)$ are parallel to the plane.

$\therefore$ $(1,-1,2) \times (-1,0,1)$ is perp. to the plane.

$$N = \begin{vmatrix} i & j & k \\ +1 & -1 & 2 \\ -1 & 0 & 1 \end{vmatrix} = i(-1) - j(3) + k(-1)$$

$(N_1, N_2, N_3) \cdot (1,-1,2) = 0$

$(N_1, N_2, N_3) \cdot (-1,0,1) = 0$

$$= (-1, -3, -1)$$

$\tilde{N} = (1,3,1)$ is still normal to the plane.

Plane eqn: $1 \cdot (x - \overset{x_0}{2}) + 3(y - \overset{y_0}{4}) + 1(z - \overset{z_0}{1}) = 0$

$$x - 2 + 3y - 12 + z - 1 = 0$$

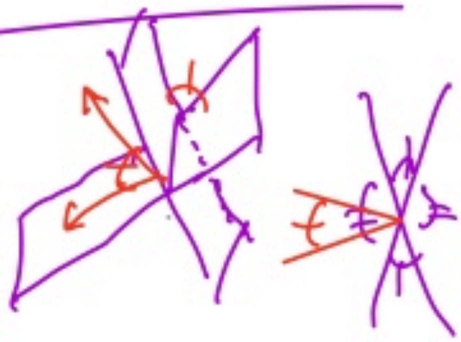
$$\boxed{x + 3y + z = 15}$$

③ Let Phil be the plane $2x - y + z = 4$

Let Paula be the plane $x - y - z = 12$

ⓐ What's the smaller angle between Phil & Paula?

ⓑ What's the equation (in parametric form) of Phil $\cap$ Paula?



ⓐ Normal vectors $(2,-1,1) = v$

$(1,-1,-1) = w$

$v \cdot w = |v|\,|w| \cos\theta$

$(2+1-1) = \sqrt{4+1+1}\sqrt{1+1+1}\cos\theta$

$2 = \underbrace{\sqrt{6}\sqrt{3}}_{3\sqrt{2}}\cos\theta \Rightarrow \cos\theta = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}.$

$$\boxed{\theta = \arccos\left(\frac{\sqrt{2}}{3}\right)}$$

the smaller angle.

bigger angle: $\pi - \arccos\left(\frac{\sqrt{2}}{3}\right)$

$= \arccos\left(-\frac{\sqrt{2}}{3}\right)$

ⓑ Eqn of Phil $\cap$ Paula

$2x - y + z = 4$

$x - y - z = 12$

Need: • pt on the line.

• velocity vector

pt: let $x=0$

$$\begin{aligned} -y+z&=4 \\ -y-z&=12 \\ \hline \text{Add} \quad -2y&=16 \Rightarrow y=-8 \end{aligned}$$

pt. $(0,-8,-4)$ on intersection

$8+z=4$
$z=-4$

Velocity vector is parallel to both planes

$\Longleftrightarrow$ perp. to both normal vectors.

$\therefore$ cross product of $(2,-1,1)$ & $(1,-1,-1)$ will give a vector in that direction.

$$\begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = i(+2) - j(-3) + k(-1)$$
$$= (2,3,-1)$$

$\therefore$ $r(t) = (0,-8,-4) + t(2,3,-1)$

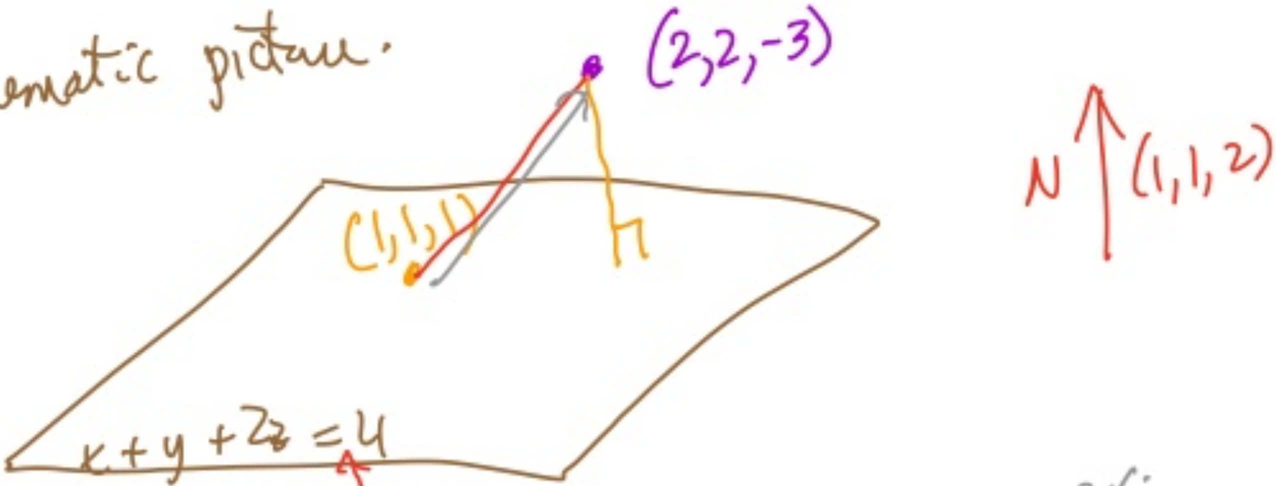
is equation of Phil $\cap$ Paula.

$$r(t) = (2t, -8+3t, -4-t).$$

---

(4) Find the distance between the point $(2,2,-3)$ and the plane $x+y+2z=4$

Schematic picture.



$B\cos\theta = \text{answer.}$

$\vec{B} = (1,1,1) - (2,2,-3)$

$= (-1,-1,4)$

$|B| = |\vec{B}| = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

$B \cdot N = |B|\,|N| \cos\theta$

$(-1,-1,4)\cdot(1,1,2) = (3\sqrt{2})(\underbrace{\sqrt{1+1+4}}_{\sqrt{6}}) \cos\theta$

$$\Rightarrow \quad 6 = \underbrace{3\sqrt{2}\underbrace{\sqrt{6}}_{\sqrt{2}\sqrt{3}}}_{6\sqrt{3}} \cos\theta$$

$\frac{1}{\sqrt{3}} = \cos\theta \qquad \theta = \arccos\left(\frac{1}{\sqrt{3}}\right)$

Ans $|B|\cos\theta = 3\sqrt{2}\,\frac{1}{\sqrt{3}} = \boxed{\sqrt{6}}$.